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A net volume equation for northeastern Minnesota

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A NET VOLUME EQUATION FOR NORTHEASTERN MINNESOTA

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A net merchantable tree volume equation and coefficients were developed for the Aspen-Birch Unit of northeastern Minnesota as part of the 1977 Minnesota Forest Inventory (fig. 1). The equation is suitable for use on digital computers by foresters, land managers, researchers, and others in need of net volume-per-tree estimates over large forest areas. Both board foot and cubic foot volume can be estimated for most tree classes.

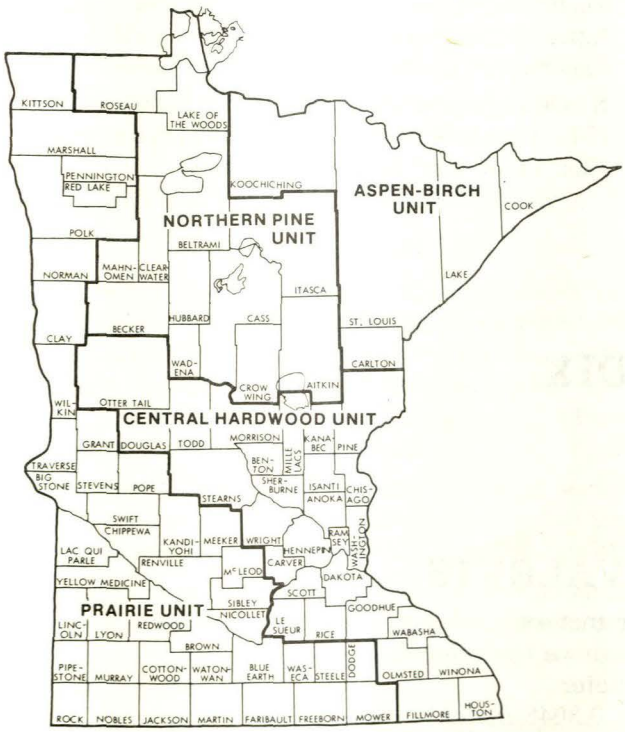

Figure 1.—*Forest Inventory Units for Minnesota.*

HOW THE EQUATION AND COEFFICIENTS WERE DEVELOPED

Equations developed by Robert N. Stone (see Appendix) based on Gevorkiantz and Olsen's (1955) composite volume tables for the Lake States were used to estimate the cubic foot volume in each tree. Stone's equations computed gross volume in a tree from the three measurements—diameter breast height (d.b.h.), merchantable height, and diameter outside bark at merchantable height (t.d.o.b.). This estimated volume was then corrected for differences in bark thickness between species and the field estimate of cull volume was subtracted to arrive at net volume (see Appendix for Stone's equation).

Experience in working with volume-d.b.h. relations suggests the following equation for this type of data (Hahn 1975, 1976):

$$V = A \cdot SI \cdot (1 - e^{-B \cdot DBH})^C$$

Where the dependent variable V is net volume of the tree, and the independent variables SI and DBH are plot site index and diameter at breast height, respectively. The value of e is the constant base of the natural logarithm (2.71828). A, B, and C are the equation coefficients. The upper asympote, A•SI, is a function of site index.

The equation coefficients were developed using data collected by field crews on 18,158 trees in the Aspen-Birch Unit. Data included d.b.h., merchantable height, t.d.o.b., site index, and tree class. For sawtimber-size trees, measurements were taken at both the sawtimber and pulpwood limits of merchantability.[1] Field crews also estimated the volume of board foot and cubic foot cull in each tree (USDA Forest Service 1975).

[1]*The pulpwood merchantability limits include trees 5 inches d.b.h. and over, from a 1-foot stump to a minimum of 4-inch t.d.o.b., or to the point where the central stem breaks into limbs. Dimensions for sawtimber are 9-inch d.b.h. to a minimum 7-inch t.d.o.b. and 11-inch d.b.h. to a minimum 9-inch t.d.o.b. for softwoods and hardwoods, respectively (USDA Forest Service 1975).*

Equation coefficients were developed for each of five tree classes: "desirable," "acceptable," "rough," "rotten," and "short-log" (see Appendix). Equation coefficients were also developed for board foot[2] and cubic foot volume (79 cubic feet per cord).

The groupings of species for the equations were primarily determined by the number of observations and the similarity of bole form among species. Species for which no observations were available were assigned equations on the basis of bole form. (For species and group numbers see Appendix.)

A computerized nonlinear regression program was used to fit the above equation to the tree data. The resulting coefficients and statistics for each regression are given in tables 1-10. Note the standard error is the standard error of the regression.

Since the average site index may be used when site index is unknown, the only data needed to determine tree volumes using this model are species, d.b.h., and tree class. Most trees in Minnesota are classed as "acceptable," with very few "desirable" trees; therefore, the person wishing to use only one equation for woods-run material should use the equation for "acceptable" trees. Net volume per tree is highly variable therefore these equations should be applied to an adequate sample over large forest areas.

HOW TO USE THE EQUATION

1. Obtain the species group number from the Appendix.
2. Select the appropriate coefficients from tables 1-10.
3. Use these coefficients in the above equation.

LITERATURE CITED

Gevorkiantz, S. R., and L. R. Olsen. 1955. Composite volume tables for timber and their application in the Lake States. Lake States Technical Bulletin 1104, 51 p.

Hahn, Jerold T. 1975. Local net volume equations for Missouri. U.S. Department of Agriculture Forest Service, General Technical Report NC-15, 8 p. U.S. Department of Agriculture Forest Service, North Central Forest Experiment Station, St. Paul, Minnesota.

Hahn, Jerold T. 1976. A local net volume equation for Iowa. U.S. Department of Agriculture Forest Service, Research Note NC-199, 4 p. U.S. Department of Agriculture Forest Service, North Central Forest Experiment Station, St. Paul, Minnesota.

U.S. Department of Agriculture, Forest Service. 1975. Forest Service Handbook. Supplement FSH-4809.11—Forest Survey Handbook.

APPENDIX

METRIC EQUIVALENTS

1,000 board feet = 3.48 cubic meters.
Breast height = 1.37 meters above the ground.
1 cubic foot = 0.0283 cubic meter.
1 foot = 30.48 centimeters or 0.3048 meter.
1 inch = 25.4 millimeters or 2.54 centimeters or 0.0254 meter.

[2]*International ¼-inch rule.*

VOLUME EQUATIONS

Stone's cubic foot volume equation uses the following independent variables: diameter at breast height (D), merchantable height (H), and diameter outside bark at merchantable height (T) to estimate cubic foot volume (CV).

$$CV = (3.0886 \times 10^{-3} + S1 + S2 + S3 + S4 + S5 + S6 + S7 + S8) \cdot 79.0$$

where

$$S1 = (2.0355 \times 10^{-3})D$$
$$S2 = (-3.0018 \times 10^{-3})T$$
$$S3 = (6.2381 \times 10^{-5})D^2$$
$$S4 = (2.5705 \times 10^{-5})D^2 \cdot H$$
$$S5 = (-7.009 \times 10^{-6})H^2$$
$$S6 = (3.6708 \times 10^{-5})H \cdot T^2$$
$$S7 = (8.14 \times 10^{-10})D^2 \cdot H^3$$
$$S8 = (-1.9 \times 10^{-9})D^2 \cdot H^2 \cdot T$$

Stone's board foot volume equation as modified by Hahn (1975, 1976) is as follows:

$$BV = 17.7488 + 7.3846 \cdot CV - 2.3523 \cdot D - 0.89945 \cdot H + 2.0726 \cdot T$$

where

BV = board foot volume.

DEFINITION OF TERMS

(USDA Forest Service 1975)

Desirable trees.—Live trees that have no serious defects that limit present or prospective use and that have relatively high vigor and contain no pathogens that may result in death or serious deterioration before rotation age. They would be favored by forest managers in silvicultural operations.

Acceptable trees.—Live trees having no serious defects that limit present or prospective use but that have pathogens or damage that may affect quality.

Rough trees.—Live trees that do not contain at least one merchantable 12-foot saw log, now or prospectively, because of roughness, poor form, or noncommercial species.

Rotten trees.—"Rough" trees in which more than 50 percent of the cull volume is rotten.

Short-log trees.—Live sawtimber sized trees that contain at least one merchantable 8- to 11-foot saw log but not a 12-foot saw log, now or prospectively.

SPECIES AND SPECIES GROUPS FOR MINNESOTA TREES

Number	Species or species group	Scientific name
SOFTWOODS		
1	White pine	*Pinus strobus*
2	Red pine	*Pinus resinosa*
3	Jack pine	*Pinus banksiana*
4	White spruce	*Picea glauca*
5	Black spruce	*Picea mariana*
6	Balsam fir	*Abies balsamea* var. *balsamea*
7	Tamarack	*Larix laricina*
8	Northern white-cedar	*Thuja occidentalis*
9	Other softwoods	*Pinus sylvestris* *Juniperus virginiana*
HARDWOODS		
10	White oak	*Quercus alba* *Quercus bicolor* *Quercus macrocarpa*
11	Select red oak	*Quercus rubra*
12	Other red oak	*Quercus ellipsoidalis*
13	Hickory	*Carya cordiformis* *Carya ovata*
14	Yellow birch	*Betula alleghaniensis*
15	Hard maple	*Acer nigrum* *Acer saccharum*
16	Soft maple	*Acer rubrum* var. *rubrum* *Acer saccharinum*
17	Ash	*Fraxinus americana* *Fraxinus nigra* *Fraxinus pennsylvanica*
18	Balsam poplar	*Populus balsamifera*
19	Paper birch	*Betula papyrifera* var. *papyrifera*
20	Bigtooth aspen	*Populus grandidentata*
21	Quaking aspen	*Populus tremuloides*
22	Basswood	*Tilia americana*
23	Elm	*Ulmus americana* *Ulmus rubra* *Ulmus thomasii*
24	Select hardwoods	*Juglans cinerea* *Juglans nigra* *Prunus serotina*
25	Other hardwoods	*Acer negundo* *Betula nigra* *Celtis occidentalis* *Populus deltoides* *Salix nigra*
26	Noncommercial species	

Table 1.—Net cubic foot volume regression coefficients for desirable trees, Aspen-Birch Unit, Minnesota, 1977

Species group number	Coefficient			Average Site index	Range of data		Number observations	Standard error (cu. ft.)
	A	B	C		Site index	D.b.h.		
1-3(9)[1]	2.154×10^{3}	0.00651	3.52	53.5	22-80	5.2-32.0	37	19.8
5(4)[1]	6.400×10^{1}	.00460	1.78	37.6	25-86	5.0-18.6	32	2.8
6-8	9.152×10^{-1}	.24112	11.45	46.4	16-82	5.2-20.4	36	3.3
14,16,17,19,22,23, (10-13, 15,24,25)[1]	$8.300 \times 10^{1}_{2}$	.00830	2.44	57.0	25-95	5.0-22.0	102	3.6
18,20,21	4.177×10^{2}	.00492	2.59	68.7	29-98	5.2-19.7	218	3.4

[1]Data were not available for species groups in parentheses.

Table 2.—Net cubic foot volume regression coefficients for acceptable trees, Aspen-Birch Unit, Minnesota, 1977

Species group number	Coefficient			Average Site index	Range of data		Number observations	Standard error (cu. ft.)
	A	B	C		Site index	D.b.h.		
1	$2.897 \times 10^{9}_{0}$	0.00005	3.13	52.6	22-99	5.2-35.9	197	28.1
2	$1.665 \times 10^{0}_{3}$	.09460	4.23	55.7	37-75	5.0-23.0	249	4.9
3(9)[1]	$3.747 \times 10^{3}_{3}$	.00133	2.31	56.9	26-80	5.0-18.1	330	2.8
4	$1.421 \times 10^{6}_{5}$	.00233	2.38	57.1	22-91	5.0-25.0	321	6.8
5	$2.036 \times 10^{5}_{0}$	.00023	2.61	39.3	16-86	5.0-19.8	1,479	3.1
6	$2.422 \times 10^{0}_{0}$	.00026	2.40	56.4	18-98	5.0-19.8	2,163	2.0
7	$1.304 \times 10^{0}_{1}$	.10503	4.06	38.9	16-75	5.0-15.1	571	1.9
8	$1.664 \times 10^{1}_{5}$	.09093	4.29	33.7	16-89	5.0-25.7	603	4.4
10,11(12,13,24,25)[1]	$6.232 \times 10^{5}_{5}$	.01262	2.88	59.0	36-81	5.0-15.8	41	1.8
14,19	$1.874 \times 10^{5}_{6}$	.00021	2.27	59.0	19-99	5.0-23.0	1,912	2.8
15	$1.366 \times 10^{6}_{5}$	.00019	2.21	59.3	46-99	5.0-18.4	162	1.6
16	$7.996 \times 10^{5}_{3}$	.00014	2.73	60.2	35-91	5.0-15.2	152	1.6
17	$3.122 \times 10^{3}_{5}$	.00026	2.44	52.0	18-99	5.0-23.6	603	3.1
18	$5.663 \times 10^{5}_{5}$	.00102	2.30	64.9	26-99	5.0-23.0	897	2.8
20	$5.328 \times 10^{5}_{3}$	.00031	2.63	63.2	35-91	5.0-22.8	131	4.2
21	$1.617 \times 10^{3}_{4}$	.00020	2.23	66.6	23-99	5.0-24.9	3,478	3.3
22	1.790×10^{4}	.00170	2.30	59.2	36-99	5.0-20.2	131	3.2
23	6.920×10^{4}	.00052	2.50	59.1	40-98	5.1-24.0	155	6.1

[1]Data were not available for species groups in parentheses.

Table 3.—Net cubic foot volume regression coefficients for rough and short-log trees, Aspen-Birch Unit, Minnesota, 1977

Species group number	Coefficient			Average Site index	Range of data		Number observations	Standard error (cu. ft.)
	A	B	C		Site index	D.b.h.		
1-3,9	2.193×10^{3}	0.00133	2.23	54.8	33-75	5.1-18.5	52	2.6
4,5	5.289×10^{-1}	.14938	4.67	39.7	20-78	5.0-13.4	63	1.8
6	6.705×10^{-1}	.08917	3.03	52.6	19-82	5.0-16.2	135	1.7
7	7.936×10^{-1}	.09784	2.96	40.7	19-68	5.0-16.7	47	2.6
8	8.175×10^{-1}	.12157	5.24	36.3	16-98	5.0-19.0	209	4.0
11,15,24-26 (10,12,13)[1]	9.267×10^{-1}	.11819	5.80	58.0	38-77	5.0-21.9	84	1.7
14,19	6.964×10^{-1}	.13971	5.68	57.9	21-99	5.0-22.3	276	2.9
16	7.831×10^{-1}	.12297	5.74	61.2	38-87	5.0-20.6	92	1.7
17	1.636×10^{0}	.10525	6.01	48.0	29-75	5.0-13.6	61	1.7
18,20,21	6.407×10^{-1}	.13405	5.60	65.6	29-99	5.0-23.7	463	3.4
22,23	6.199×10^{0}	.06672	6.82	54.8	40-85	5.0-40.2	44	5.2

[1]Data were not available for species groups in parentheses.

Table 4.—Net cubic foot volume regression coefficients for rotten trees, Aspen-Birch Unit, Minnesota, 1977

Species group number	Coefficient			Average Site index	Range of data		Number observations	Standard error (cu. ft.)
	A	B	C		Site index	D.b.h.		
1,3-5(2,9)[1]	3.084×10^{0}	0.05940	5.30	43.2	22-70	5.2-21.4	31	4.5
6,7	5.590×10^{-1}	.10190	4.76	49.0	18-70	5.0-14.7	79	3.4
8	4.386×10^{-1}	.07417	3.52	37.2	16-98	5.0-27.3	270	4.6
10,11,15,24,25 (12,13,26)[1]	2.458×10^{0}	.05138	6.01	55.5	36-77	5.1-25.3	98	8.0
14	1.502×10^{-1}	.10573	6.31	55.6	25-75	5.8-28.1	55	7.1
16	6.660×10^{0}	.04680	5.61	58.0	38-87	5.0-21.8	1,142	4.3
17	7.845×10^{-2}	.19333	4.04	48.4	25-82	5.0-22.9	46	4.2
18,20,21	4.962×10^{-1}	.03528	1.73	63.7	29-99	5.0-28.9	694	6.9
19	1.606×10^{-1}	.17045	5.80	51.1	22-83	5.0-27.5	171	5.7
22,23	1.445×10^{-1}	.18231	5.94	56.1	35-82	5.1-34.4	46	7.5

[1]Data were not available for species groups in parentheses.

Table 5.—Net board foot[1] volume regression coefficients for desirable trees, Aspen-Birch Unit, Minnesota, 1977

Species group number	Coefficient			Average Site index	Range of data		Number observations	Standard error (bd. ft.)
	A	B	C		Site index	D.b.h.		
1-3,5,7,8(4,6,9)[2]	3.643×10^{5}	0.00087	2.61	49.7	16-86	9.6-32.0	50	165.5
14,16,17,19,22,23 (10,13,15,24,25)[2]	2.046×10^{3}	.00336	2.23	57.8	25-82	11.2-22.0	40	41.9
18,20,21	2.064×10^{3}	.00337	2.24	68.7	36-98	11.0-19.7	76	34.7

[1]International ¼-inch rule.
[2]Data were not available for species groups in parentheses.

Table 6.—Net board foot[1] volume regression coefficients for acceptable trees, Aspen-Birch Unit, Minnesota, 1977

Species group number	Coefficient			Average Site index	Range of data		Number observations	Standard error (bd. ft.)
	A	B	C		Site index	D.b.h.		
1	8.873×10^{4}	0.00130	2.62	52.6	22-99	9.3-35.9	178	207.3
2	1.614×10^{4}	.00054	1.80	54.4	37-75	9.1-23.0	162	36.5
3(9)[2]	5.018×10^{3}	.00261	2.40	58.1	33-80	9.1-18.1	163	21.6
4	2.379×10^{6}	.00060	2.96	57.3	22-91	9.0-25.0	210	47.8
5	3.544×10^{6}	.00069	3.03	44.0	16-86	9.0-19.8	195	39.7
6,7	3.989×10^{3}	.00233	2.26	56.3	16-88	9.0-19.8	554	20.5
8	3.869×10^{3}	.00251	2.21	34.7	16-89	9.0-25.7	680	39.7
10,11,15,16 (12,13,24,25)[2]	2.180×10^{4}	.00037	1.80	58.5	43-82	11.0-18.4	53	24.6
14,19	7.328×10^{3}	.00051	1.68	57.4	19-99	11.0-23.0	253	39.6
17	8.908×10^{3}	.00048	1.65	50.1	21-99	11.0-23.6	99	48.2
18	4.059×10^{5}	.00019	2.08	64.5	26-98	11.0-23.0	251	33.5
20,21	9.322×10^{2}	.00566	2.43	68.1	23-99	11.0-24.9	1,009	40.1
22	7.791×10^{2}	.00548	2.35	58.3	40-83	11.0-20.2	38	45.5
23	1.007×10^{5}	.00110	2.63	56.6	40-84	11.0-24.0	94	51.0

[1]International ¼-inch rule.
[2]Data were not available for species groups in parentheses.

Table 7.—Net board foot[1] volume regression coefficients for short-log trees, Aspen-Birch Unit, Minnesota, 1977

Species group number	Coefficient			Average Site index	Range of data		Number observations	Standard error (bd. ft.)
	A	B	C		Site index	D.b.h.		
1-7(9)[2]	5.502×10^{-1}	0.10168	4.93	52.6	35-82	9.0-16.7	50	37.5
8	2.445×10^{0}	.06232	1.71	38.9	20-98	9.0-19.0	55	29.6
11,14,16,17,19,22,23 (10,12,13,15,24-26)[2]	1.846×10^{1}	.05735	5.02	57.6	38-83	11.0-40.2	41	40.1
18,20,21	2.963×10^{5}	.00014	2.06	67.7	36-99	11.1-20.0	70	31.1

[1]International ¼-inch rule.
[2]Data were not available for species groups in parentheses.

Table 8.—Net cubic foot volume in saw log regression coefficients for desirable trees, Aspen-Birch Unit, Minnesota, 1977

Species group number	Coefficient			Average Site index	Range of data		Number observations	Standard error (cu. ft.)
	A	B	C		Site index	D.b.h.		
1-3,5,7,8 (4,6,9)[1]	1.056×10^{4}	0.00148	2.53	49.7	16-86	9.6-32.0	50	23.9
14,16,17,19,22,23 (10-13,15,24,25)[1]	2.200×10^{3}	.00158	2.31	57.8	25-82	11.2-22.0	40	6.3
18,20,21	3.049×10^{3}	.00185	2.49	68.7	36-98	11.0-19.7	76	5.5

[1]Data were not available for species groups in parentheses.

Table 9.—Net cubic foot volume in saw log regression coefficients for acceptable trees, Aspen-Birch Unit, Minnesota, 1977

Species group number	Coefficient			Average Site index	Range of data		Number observations	Standard error (cu. ft.)
	A	B	C		Site index	D.b.h.		
1	3.472×10^{3}	0.00191	2.50	52.6	22-99	9.3-35.9	178	29.7
2	4.255×10^{3}	.00028	1.66	54.4	37-75	9.1-23.0	162	6.1
3(9)[1]	2.275×10^{3}	.00170	2.38	58.1	33-80	9.1-18.1	163	3.7
4	1.550×10^{3}	.00064	1.78	57.3	22-91	9.0-25.0	210	8.2
5	1.663×10^{3}	.00149	2.14	44.0	16-86	9.0-19.8	195	7.2
6,7	1.756×10^{3}	.00147	2.25	56.3	16-88	9.0-19.8	554	3.6
8	1.521×10^{3}	.00141	2.10	34.7	16-89	9.0-25.7	680	6.8
10,11,15,16 (12,13,24,25)[1]	5.096×10^{3}	.00015	1.60	58.5	43-82	11.0-18.4	53	3.5
14,19	1.187×10^{3}	.00013	1.32	57.4	19-99	11.0-23.0	253	6.7
17	2.457×10^{3}	.00019	1.48	50.1	21-99	11.0-23.6	99	8.8
18	1.584×10^{2}	.00015	1.40	64.5	26-98	11.0-23.0	251	6.1
20,21	3.185×10^{2}	.00303	2.20	68.1	23-99	11.0-24.9	1,009	5.9
22	2.629×10^{2}	.00284	2.10	58.3	40-83	11.0-20.2	38	6.0
23	3.435×10^{4}	.00054	2.40	56.6	40-84	11.0-24.0	94	8.0

[1]Data were not available for species groups in parentheses.

Table 10.—Net cubic foot volume in saw log regression coefficients for short-log trees, Aspen-Birch Unit, Minnesota, 1977

Species group number	Coefficient			Average Site index	Range of data		Number observations	Standard error (cu. ft.)
	A	B	C		Site index	D.b.h.		
1-7(9)[1]	6.107×10^{2}	0.00194	2.21	52.6	35-82	9.0-16.7	50	6.3
8	1.071×10^{2}	.00146	1.59	38.9	20-98	9.0-19.0	55	6.0
11,14,16,17,19,22,23 (10,12,13,15,24-26)[1]	4.456×10^{2}	.00228	2.27	57.6	38-83	11.0-40.2	41	4.4
18,20,21	5.795×10^{1}	.00092	1.33	67.7	36-99	11.1-20.0	70	4.9

[1]Data were not available for species groups in parentheses.

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Describes a net volume equation for northeastern Minnesota developed as part of the 1977 Minnesota Forest Inventory. Equation coefficients are presented by species groupings for both cubic foot and board foot volumes for five tree classes.

KEY WORDS: inventory, computer, growing-stock, cull, cubic foot volume, board foot volume.

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